Homework #3

due Tuesday, 10/11/11, in class

Problem 1 Consider the following simultaneous-move game:

		Player 2	
		Y	Z
Player 1	W	a,b	c,d
	X	e,f	g,h

a. List all inequalities that must hold for (W, Y) to be a dominant strategy equilibrium. (hint: an example of an inequality would be $g \geq h$)

b. List all inequalities that must hold for (W, Y) to be a Nash equilibrium.

Problem 2 Consider the following simultaneous-move game:

		2		
		X	Y	Z
1	A	4,4	10,1	0,3
	B	1,10	4,4	9,7
	C	3,0	7,9	4,4

a. Find the pure strategy Nash equilibria, if any.

b. There is a mixed strategy equilibrium where player 1 randomizes between A and C, and where player 2 randomizes between X and Z. Solve for this equilibrium

c. (optional, 5 points extra credit) There is a mixed-strategy equilibrium where both players randomize between all three strategies. Solve for this equilibrium.

Problem 3 This problem demonstrates a seeming peculiarity about mixed strategy Nash equilibria. Consider the following game between the Chicago Bears' offense and the Detroit Lions' defense. Payoffs are the number of yards advanced (positive yards for Chicago are negative yards for Detroit).

		Detroit	
		run defense	pass defense
Chicago	run	-2,2	5,-5
	pass	15,-15	1,1

a. Find all pure strategy Nash equilibria, if any. Then find the mixed-strategy Nash equilibrium of the game.

b. Now suppose that the Bears improve their run game by bringing Mike Ditka[1] out of retirement:

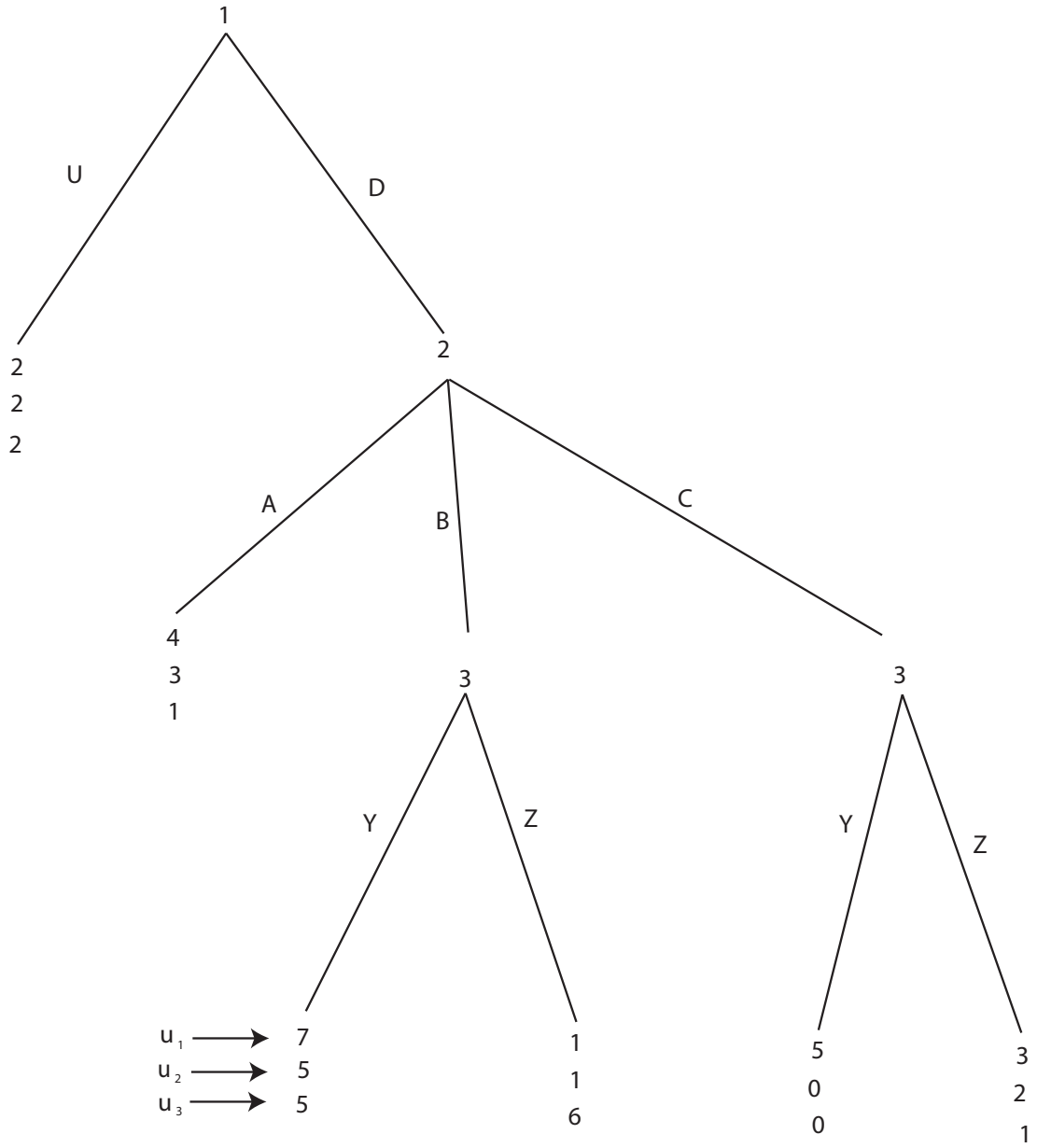
[1] While Ditka played tight end, the combination of his blocking and the downfield threat he poses as a receiver, even at 71, would help their running game immeasurably.

		Detroit	
		run defense	pass defense
Chicago	run	-2,2	10,-10
	pass	15,-15	1,1

Find the mixed-strategy Nash equilibrium of the new game.

c. When running the football becomes a more attractive option for the Bears, do they run more often, or pass more often? Can you explain why?

Problem 4 Consider the sequential move game below:



What is the equilibrium outcome of this game?

Problem 5 Hippos are, from birth, either hawkish or dovish. Hawkish hippos are more likely to have hawkish babies, and dovish hippos are more likely to have dovish babies, though there is, of course, some chance of a parent of one type having a child of the other. Several times a day, an hippo comes across another hippo, and both hippos benefit/are harmed by the interaction as follows:

		hippo 2	
		Hawk	Dove
hippo 1	Hawk	-12,-12	9,0
	Dove	0,9	7,7

Think of the payoffs here as gain/loss to reproductive fitness. High payoffs are correlated with high fertility, low payoffs with low fertility.

a. Suppose most hippos are hawks. Will hawks or doves reproduce more? Explain why.

b. Suppose most hippos are doves. Will hawks or doves reproduce more? Explain why.

c. Solve for the fraction of hawkish hippos that would result in a stable Nash equilibrium, meaning that hawks and doves reproduce at the same rate.

d. Explain how an evolutionary process would result in the stable outcome you identify in part c.

e. Suppose that over time, hippos evolve shorter teeth and claws, so that the payoffs in the (Hawk, Hawk) box change from $(-12, -12)$ to $(-5, -5)$. All other payoffs are the same. Is this adaptation likely to benefit or harm the hippo population? (Hint: is the average fitness of an hippo higher or lower once a new equilibrium between hawks and doves evolves?)

Problem 6 Two bills are being considered in Congress (bill A, which would reinstitute the Volstead Act, and bill B, which would prohibit anyone of Canadian origin from owning property). Here are the payoffs to Congress and the president depending upon which laws are passed:

Outcome	Congress	President
Bill A only	8	-1
Bill B only	-1	9
Both bills	5	5
Neither bill	0	0

a. Suppose that Congress first decides which of the four options to select. The president can then either sign or *veto*, in which case no law is passed. Which bills become laws in the equilibrium of this sequential game? Explain, with aid of a diagram.

b. Now suppose that the president has a *line-item veto*, so that if Congress passes both bills, he can choose to sign bill A or bill B only. However, he cannot enact laws that Congress does not pass. Which bills become laws in the equilibrium of this game? Explain.

c. It is often suggested that giving the president a line-item veto would be a good way to make government work more efficiently, as then he would not have to veto entire bills just because he felt one provision of the bill would make a bad law. In light of this question, what do you think of this suggestion?