Homework 2

due Monday, September 24 by noon

Instructions: Complete all 8 problems. Answers may be handwritten or typed. Students may work together, but must independently write their own answers. Failure to do so will result in a grade of zero. Please turn in your homework to me or under the door of my office (335L) by noon on the assigned due date.

Problem 1 Suppose that most people will not speed if the expected fine is greater than $200. Given current police practices, the probability of being caught speeding is 20%.

a. How high must the actual fine for speeding (what you have to pay if you get a ticket) be to deter most people from speeding?

b. Suppose the Lexington mayor caps the amount police can fine speeding motorists at $500. The police vow to step up enforcement in order to cintinue to deter speeding. How high must the probability of being caught speeding be in order to deter most people from speeding?

Problem 2 Suppose an investor is considering a business opportunity that would cost $100, but which would generate a return according to the probability distribution below:

Probability	Return
0.2	$100
0.3	$30
0.2	-$10
0.3	-$30

a. What is the expected return of the uncertain investment? What is the variance? What is the standard deviation?

b. Would a risk neutral investor take this business opportunity, if he has no alternative investment in mind? What about a risk averse investor?

Problem 3 Answer the following questions about expected utility and risk aversion:

a. True/false/uncertain: Rex, who is risk averse, prefers a salaried job that will pay him $100,000 with certainty to a commission-based job that will pay him $20,000 with probability .5 and $200,000 with probability .5. (Explain why you think it is true, false, or uncertain.)

b. Kyle is an English PhD student; there is some chance he will get a prestigious university job that will pay him $80,000/year (probability p). If he does not get this job, he will have to take a job at Starbuck's, paying $25,000/year (probability $(1 - p)$). Kyle's utility function over salary is given by $\sqrt{w}$, where w is the amount of his salary. Before Kyle has a chance to finish his PhD, he is offered a job at Dunder Mifflin paying $60,000/year. How low does p have to be before Kyle is better off taking the Dunder Mifflin job? (hint: p is some number between 0 and 1. For example, $p = .25$ means there is a 25% chance of Kyle getting the university job and a 75% chance of his working at Starbuck's.)

Problem 3 A driver faces a 5% probability that his car will be in an accident and will be worth nothing. Consider three drivers with cars that have value $30,000. Abdulla's utility function over the value of his car W is $u(W) = ln(1 + W)$. Bedriya's utility function is $u(W) = 100 + 0.5W$.

a.   What is Abdulla's risk premium? (hint: first, calculate Abdulla's expected utility. Second, calculate X, Abdulla's certainty equivalent. Third, calculate the expected dollar value of the car without insurance. Fourth, calculate risk premium, the difference between X and the expected dollar value of the car)

b.   What is Bedriya's risk premium?

d.   Which of these two people is less likely to take on risk? Which is more likely? How do you know?

Problem 4 The marginal product of labor in the production of computer chips is 50 chips per hour. The marginal rate of of technical substitution of labor for capital is $\frac{1}{4}$. What is the marginal product of capital?

Problem 5 The production function for the bicycles of BIKE, Inc. is given by:

$$Q = 10K^{.5}L^{.5}$$

where Q is the number of bicycles produced per day, K is hours of machine time, and L is hours of labor input. BIKE's competitor, PEDAL, Inc., is using the production function:

$$Q = 10K^{.6}L^{.4} \tag{1}$$

a.   If both BIKE and PEDAL an equal amount of labor and capital (say, $L = K = X$, where X is any number), which firm will generate more output?

b.   Assume that, for both BIKE and PEDAL, capital is limited to 9 machine hours, but labor is unlimited in supply. In which company is the marginal product of labor greater? Explain.

Problem 6   Suppose a chair manufacturer is producing in the short run (with its existing plant and equipment). The manufacturer has observed the following levels of production corresponding to different numbers of workers:

# of workers	# of chairs
1	10
2	18
3	24
4	28
5	30
6	28
7	25

a.   Calculate the marginal and average product of labor for this production function.

b.   Does this production function exhibit diminishing returns to labor? Explain.

c.   Explain intuitively what might cause the marginal product of labor to become negative.

Problem 7 The long-run production function for a firm's product is given by $q = f(K, L) = 5 * K * L$. The price of capital is $10 and the price of labor is $15.

a.   Suppose the firm wishes to produce output of 500. List 5 combinations of capital and labor that the firm can transform into 500 output.

b.   For each of your 5 combinations from part a, give the cost of using that combination of capital and labor. Which is the lowest?

c.   For your lowest cost combination from part b, calculate the marginal product of capital (MPK) and the marginal product of labor (MPL).

d.   For your answer in parts b-c, is your marginal product per dollar equal across the two inputs? If not, should the firm use more labor-intensive production or more capital-intensive production?

Problem 8 A firm has production function $f(K, L) = \sqrt{KL}$. In the short run, the firm has capital $K = 400$; this cannot be changed in the near future. The cost of a unit of capital is \$20, while the cost of a unit of labor is \$30.

a.   In the short-run, where capital is fixed at $K = 400$, how much labor does the firm need to employ in order to produce $q = 300$ output? What is the cost of producing 300 output?

b.   Repeat part a for $q = 400$, $q = 500$ and $q = 600$.

c.   What is the cost of producing q output in the short-run?

d.   What is the marginal cost of producing a 301st unit? A 401st unit? If you are comfortable doing so, you may answer this question by writing down the marginal cost function directly, rather than recalculating total cost for $q = 301$ and $q = 401$.

e.   Given your answer to part c, draw a graph with the firm's average total cost, average variable cost, and marginal cost (hint: $MC = \frac{3}{20}q$).

f.   Suppose the firm operates in a competitive market, and the price of the output good is $p = \$15$. How much output will the firm supply. What will the firm's profit be at this price?

g.   Now suppose that the competitive price changes, to \$p. How much output will the firm supply, as a function of p? What is the minimum price that the firm needs in order to be profitable in the short-run? In the long-run?

h.   Finally, suppose now the firm's marginal cost is still $MC = \frac{3}{20}q$, but the firm is a monopoly with demand curve equal to $p = 24 - \frac{3}{40}q$. Solve for the firm's profit-maximizing price and quantity. What is the monopolist's profit?