## Surplus final questions

Exam directions: answer 7 of the 8 equally-weighted questions. Provide detailed explanations for your answers; unsupported answers are unlikely to receive points. You may use calculators, books, and notes, but no laptops, cell phones, or other communication devices. You may ask the proctor about specific calculations, but he will not help you with the content of the exam.

**Problem 1** A town has 100 voters: 51 conservatives and 49 liberals. A conservative and a liberal candidate are running in an election. Voting is by simple majority. Each conservative gets a payoff of 10 if the conservative is elected and -10 if the liberal is elected; each liberal gets 10 if the liberal is elected, and -10 if the conservative is elected. It costs each citizen 1 to vote (so, for example, the total payoff to a conservative from voting for a victorious conservative candidate is 9).

- a. Explain why it is not a Nash equilibrium for everybody to vote.
- b. Explain why it is not a Nash equilibrium for nobody to vote.
- c. Is it a Nash equilibrium for 50 conservatives and 49 liberals to vote?

**Problem 2** Consider a market with demand p = 9 - Q, and with two firms engaged in quantity competition; that is, firm 1 names a quantity  $q_1$  and firm 2 names a quantity  $q_2$ , and both firms sell their products at price p = 13 - Q, where  $Q = q_1 + q_2$ .

Suppose both firm 1 and firm 2 have marginal costs of 1.

- a. If the two firms set quantity simultaneously, what are the equilibrium quantities and price?
- b. How much profit does firm 1 earn in this equilibrium? Firm 2?

c. Suppose the two firms form a cartel and act as a monopolist. By how much would they be able to increase total profit? What would happen to price?

**Problem 3** Consider the following game:

		Francis	
		Left	Right
Frances	Left	$^{\mathrm{a,b}}$	c,d
	Right	e,f	g,h

What inequalities must hold for Left, Left to be a dominant strategy equilibrium? A Nash equilibrium?

**Problem 4** In the game of "take-away", there are 21 pennies on a table. You and an opponent alternatively remove the pennies from the table. The only stipulation is that on each turn you may only remove exactly one, two, or three pennies. You may not pass. The person who removes the last penny loses.

It is true that in any subgame perfect equilibrium of take-away one of the players always wins. Which one? What strategy does she play?

**Problem 5** Consider the following game; payoffs are given at the end of each branch as  $u_1, u_2$ .



Suppose you are told that in a backwards induction equilibrium of the game, player 1 plays strategy A. What must be true of the parameters c and d?

**Problem 6** Rex Banner, the star linebacker for the Moosejaw Misfits, is demanding his \$100,000 salary be doubled; he is threatening to sit out the season if it is not. The Misfits need this decided immediately, so they can sign another linebacker if Banner indeed does sit out.

A meeting is scheduled, at which the Misfits will make an offer (either agree to double, or not double) which Banner must then immediately accept or reject.

If Banner rejects the offer, he'll spend the year working at Wal~Mart, for a salary of \$20,000, and the Misfits will have to sign a new linebacker. The going rate for linebackers of Banner's quality is \$250,000. If Banner accepts the offer, his payoff is equal to his salary and the team's is the negative of the salary.

a. Draw the game tree associated with this game. Make sure to indicate the order of events, all strategies for both players, and payoffs.

b. Use backwards induction to solve the game. Is Banner's threat to sit out the season credible?

c. Suppose Banner realizes he is not in a strong bargaining position and so hires an agent to go to the meeting for him. Banner gives the agent instructions and then pays the agent \$50,000 if his instructions are followed and \$0 otherwise. Can Banner use the agent to his advantage, or is he a waste of money?