Homework 7

Problem 1 Two firms face the following payoff matrix:

		Firm 2: Low price	Firm 2: High price
Firm 1	Low price	2,0	1,2
	High price	0,7	6,6

Given these payoffs, Firm 2 wants to match Firm 1's price, but Firm 1 does not want to match Firm 2's price. What, if any, are the pure-strategy Nash equilibria of this game?

There are no pure-strategy Nash equilibria of this game. There is a mixed-strategy Nash equilibrium with Firm 1 playing 'Low price' $\frac{1}{3}$ of the time and 'High price' $\frac{2}{3}$ of the time, and Firm 2 playing 'Low price' $\frac{5}{7}$ of the time and 'High price' $\frac{2}{7}$ of the time.

Problem 2 Lori employs Max. She wants him to work hard rather than to load. She considers offering him a bonus or not giving him one. All else the same, Max prefers to loaf.

		Max: Work	Max: Loaf
Lori	Bonus	1,2	-1,3
	No bonus	3,-1	0,0

If Max and Lori choose actions simultaneously, what is the Nash equilibrium of this game?

The Nash equilibrium is 'No bonus, Loaf'. Note that this game is a prisoners dilemma.

Problem 3 Two firms are planning to sell 10 or 20 units of their goods and face the following payoff matrix:

		Firm 2: 10	Firm 2: 20
Firm 1	10	30,30	50,35
	20	40,60	20,20

a. What is their Nash equilibrium if both firms make their decisions simultaneously?

There are two pure-strategy Nash equilibria, '10,20', and '20,10'.

b. Suppose Firm 1 can decide quantity first. What is the subgame perfect Nash equilibrium?

1 produces 10, 2 produces 20 if 1 produces 10 and 10 if 1 produces 20.

c. Suppose Firm 2 can decide quantity first. What is the subgame perfect Nash equilibrium?

2 produces 10, 1 produces 20 if 2 produces 10, 1 produces 10 if 2 produces 20.

Problem 4 Consider the following game between the IRS and a professor. Suppose that the professor can choose between cheating and not cheating when filing his tax return and IRS can choose between audit and no audit. Assume that an audit costs the IRS 10 and that it increases the revenues by 20 if the professor is cheating (it gives no additional revenues if the professor is not cheating). Furthermore, if the professor doesn't cheat, he pays a tax of 30 (independently of whether there is an audit or not), while if he does cheat he pays 20 in case he is not caught. However, if there is an audit and the professor is caught (auditing is perfect in the sense that cheaters are detected with probability 1) then the professor has to pay a tax of 30 and a fine of 10 (total=40).

a. Write down the payoff matrix of the game described above.

The matric below captures the story above:

		IRS	
		audit	no audit
Professor	cheat	-40,30	-20,20
	don't cheat	-30,20	-30,30

b. Derive all Nash equilibria of the game, mixed as well as pure.

There are no pure-strategy Nash equilibria. There is one mixed equilibrium, in which the professor cheats with probability $\frac{1}{2}$, and the IRS audits with probability $\frac{1}{2}$.

Problem 5 A newspaper runs the following contest: each of 100 participants mails in a postcard on which he writes down a whole number between 0 and 1000 (inclusive). Given the entries, the *target number* is defined to be fraction $\frac{9}{10}$ multiplied by the highest entry, rounding downward if the result is not a whole number. All participants who choose the target number split a $10,000 prize.

a. Solve this game by iteratively removing dominated strategies. You should get a unique prediction for the game.

Clearly, any number over 900 could never possibly win, so we can remove these dominated numbers. Having done this, now any number over 810 cannot possibly win, so we can remove all of these numbers. This continues until all numbers greater than 0 are eliminated. Conclude the equilibrium of this game is for everyone to play 0 and split the prize.

b. If you were actually participating in this contest, what number would you play?

I will ask you for an actual number choice on the April 7th quiz.

Problem 6 Give a brief example of a 2-player prisoners dilemma from a movie or book. Explain who the players are, the situation in which they must choose to cooperate or not with each other, and what you think their payoffs are. Draw a payoff matrix associated with your example. In your example, did the players cooperate or not? Why do you think this is?