Homework #3

due 10/11/11 in class

Problem 1 Let T be some finite integer. Solve the following maximization problem:

$$\max_{\{x_t\}} \sum_{t=1}^{T} \sqrt{x_t} \quad \text{subject to} \quad \sum_{t=1}^{T} x_t \leq 1,$$
$$x_t \geq 0, \ t = 1, 2, ..., T$$

Problem 2 A consumer has income $I > 0$ and faces prices (p_1, p_2, p_3) for the three goods she consumes. Her utility function is $u(x_1, x_2, x_3) = x_1 x_2 x_3$. All goods must be consumed in nonnegative amounts. Furthermore, she must consume at least 2 units of good 2, and cannot consume more than 1 unit of good 1.

a. Assuming $I = 4$ and $(p_1, p_2, p_3) = (1, 1, 1)$, calculate her optimal consumption bundle.

b. Now assume $I = 6$ and $(p_1, p_2, p_3) = (1, 2, 3,)$. What is her optimal consumption bundle?

Problem 3 Your ship is overdue in port and the beer is running out. The remaining supplies are divided up and you get 22.5 liters. The ship will not reach port before tomorrow morning, and there is a 60% chance that it will arrive then. You can't take beer with you when you leave the ship, so you could drink it all today, to make sure it isn't wasted. On the other hand, there is a 40% chance that you will still be afloat all day tomorrow, and a 10% chance that you will be afloat the day after that. You could save some beer in case you need it for the second day, or the third. It is certain you will reach port before the fourth day.

You are an expected utility maximizer, and your utility is $6000B - 250B^2$, where B is liters of daily beer consumption. How much beer should you drink today? How much should you save for tomorrow? For the day after tomorrow? (hint: your answers should be round numbers)

Problem 4 Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = ax^2 + by^2 + 2cxy + d$. For what values of a, b, c, and d is f a concave function?

Problem 5 A firm produces an output y using two inputs x_1 and x_2 as $y = \sqrt{x_1 x_2}$. Union rules obligate the firm to use at least one unit of x_1 in its production process. The input prices of x_1 and x_2 are w_1 and w_2, respectively. Assume that the firm wishes to minimize the cost of producing $\overline{y}$ unites of output.

a. Set up the firm's cost-minimization problem. Is the feasible set closed? Compact? Convex?

b. Describe the Kuhn-Tucker first-order conditions. Are they sufficient for a solution? Why or why not?