# Homework 4

due 3/21/12

**Problem 1** Consider infinite repetitions of the games in figures 1, 2, and 3. In each case, sketch the set of payoff vectors attainable in a subgame perfect equilibrium, and those which are attainable in a subgame perfect equilibrium using Nash reversion strategies. (Suppose that the discount rate is very close to one.)

| | | 2 | |
|---|---|---|---|
| | | $A$ | $B$ |
| 1 | $a$ | $2,2$ | $1,0$ |
| | $b$ | $0,1$ | $0,0$ |

Figure 1: Normal form game 1

| | | 2 | |
|---|---|---|---|
| | | $L$ | $R$ |
| 1 | $T$ | $3,0$ | $1,-2$ |
| | $B$ | $5,4$ | $-1,6$ |

Figure 2: Normal form game 2

| | | 2 | |
|---|---|---|---|
| | | $L$ | $R$ |
| | $T$ | $2,-2$ | $4,1$ |
| 1 | $M$ | $1,3$ | $0,0$ |
| | $B$ | $5,3$ | $3,4$ |

Figure 3: Normal form game 3

**Problem 2** Consider the normal form game $G$ in figure 4 below.

**a.** Determine the set of Nash equilibria of the normal form game (note that the game is zero-sum).

**b.** Let $G^\infty(\frac{3}{4})$ be an infinite repetition of $G$ with common discount rate $\frac{3}{4}$. Sketch both the set of feasible payoffs of $G^\infty(\frac{3}{4})$ and the set of payoffs which are sustainable in some subgame perfect equilibrium of $G^\infty(\frac{3}{4})$.

**Problem 3** Consider an infinite repetition of the normal form game in figure 5. For what values of $\beta$ can the play path $\{(C,C),(C,C),...\}$ be supported in a subgame perfect equilibrium?

| | | 2 | | |
|---|---|---|---|---|
| | | $X$ | $Y$ | $Z$ |
| 1 | $A$ | $10, 0$ | $0, 10$ | $0, 10$ |
| | $B$ | $9, 1$ | $1, 9$ | $1, 9$ |
| | $C$ | $2, 8$ | $8, 2$ | $1, 9$ |
| | $D$ | $2, 8$ | $2, 8$ | $7, 3$ |
| | $E$ | $4, 6$ | $5, 5$ | $6, 4$ |

Figure 4: Normal form game 4

| | | 2 | |
|---|---|---|---|
| | | $C$ | $D$ |
| 1 | $C$ | $2, 2$ | $0, 8$ |
| | $D$ | $8, 0$ | $1, 1$ |

Figure 5: Normal form game 5

**Problem 4** You and a friend are computing the subgame perfect equilibria of an infinitely repeated game. Your friend suggests that to perform the computation efficiently, you should begin by eliminating all dominated actions from the stage game. Evaluate your friend's suggestion.

**Problem 5** MWG problem 12.AA.1

**Problem 6** MWG problem 12.B.1

**Problem 7** MWG problem 12.B.8