Merger efficiencies and price effects in differentiated Cournot oligopoly

Jeremy Sandford*

Compass Lexecon

July 6, 2022

Abstract

I derive a formula for the minimum cost savings that would offset the incentive to increase price created by a merger, when differentiated firms compete in quantities. The formula depends only on pre-merger information on margins and demand slopes, and is invariant to demand and cost curvature. The paper then develops an algorithm to infer demand slopes – and thus allow calibration of parameterized demand and cost curves – from pre-merger data. While the Cournot model of quantity competition is commonly accompanied by an assumption that rivals' products are interchangeable, the inflexibility of this assumption and its implications opens the model to criticisms. The paper examines the advantages of relaxing the assumption of interchangeability, in particular greater consistency with pre-merger data, greater scope for profitable mergers, and greater flexibility in merging firms' cost curves. An extended numerical example illustrates the application of a differentiated Cournot model to a hypothetical industry.

JEL Classification: L11, L13, L41.

Keywords: differentiated Cournot, $CMCR$, merger simulation, efficiencies

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*jsandford@compasslexecon.com, jasandford.com. I thank David Balan for helpful comments. This paper was written while I worked at the Federal Trade Commission. The opinions expressed here are those of the author and not necessarily those of Compass Lexecon, the Federal Trade Commission, or any FTC Commissioners.

1 Introduction

Consider an industry in which firms produce differentiated substitutes, and compete *a la* Cournot by choosing quantities. A merger generically changes the incentives of each merging firm, in that the firm internalizes the negative effect of additional production on the profit of its former rival, and thus optimally decreases its quantity. Should the merger reduce the costs of one or both merging firms, this incentive may be offset to some extent by the now-greater profit margin associated with each sale. In this paper, I derive the minimum cost savings that would completely offset the change in incentive created by the merger when differentiated firms compete in quantity. In keeping with the literature, I refer to this metric as Compensating Marginal Cost Reduction ("$CMCR$").

$CMCR$ depends on pre-merger margins, prices, and demand slopes, but is invariant to assumptions on curvature of demand and cost curves. While prices and margins are commonly observable by researchers and practitioners, information on demand slopes may or may not be available. Thus, I also provide a method for calibrating the necessary demand slopes from pre-merger information on margins and diversion ratios. The resulting calibration additionally enables simulation of post-merger prices based on both the calibrated demand slopes and assumptions about the curvature of demand and cost curves away from the pre-merger equilibrium.

I then use merger simulation tools to compare the differentiated Cournot model to the more commonly-used homogeneous Cournot and differentiated Bertrand models. In contrast to the homogeneous Cournot model, I show that the differentiated Cournot model can flexibly match per-merger information on margins, thus more plausibly explaining the pre-merger equilibrium. Further, mergers of differentiated Cournot firms appear to be more profitable than mergers of homogeneous Cournot firms, suggesting that the differentiated Cournot model may better capture the incentive to merge. I provide evidence that mergers of differentiated Cournot competitors can result in greater price increases than comparable mergers of Bertrand competitors, but note that this result is not universal. Finally, I provide evidence that when differentiated firms compete in quantities, capacity constraints on merging firms mitigate price effects, while capacity constraints on nonmerging firms exacerbate them, with the former effect being relatively more important.

The industrial organization literature has studied models of differentiated firms competing in quantities for decades. Singh and Vives (1984) argue that when products are differentiated, competition in quantities renders the market "more monopolistic" – and thus higher-priced – than would competition in prices. This result is generalized by Vives (1985), Okuguchi (1987), and Qiu (1997), and limited by Qiu (1997), Häckner (2000), and Alipranti et al. (2014). As noted by Vives (1985), the intuition for why Cournot industries tend to have higher prices is that when a Cournot firm sets price, it reasons that its rivals will respond to a higher price by increasing their prices (so as to maintain a constant quantity). In contrast, when a Bertand competitor sets its price, it reasons that each rival will not adjust price in response. Hence, the Cournot competitor has an additional incentive to increase price, relative to the Bertrand competitor. This intuition is further unpacked in a numerical example in section 2.

While the literature has extensively studied the relationship between mode of competition and outcome, comparatively little is known about how mode of competition and degree of differentiation affect merger outcomes. For example, Nocke and Whinston (2021) assess the effectiveness of concentration screens as described in the FTC/DOJ *Horizontal Merger Guidelines*, finding that change in the Herfindahl index is a reasonable proxy for merger price effects both when homogeneous firms compete in quantities and when differentiated firms compete in prices.

The lack of attention paid to the antitrust implications of the differentiated Cournot model in the academic literature appears to reflect the practices of antitrust practitioners. For example, the FTC employed homogeneous Cournot models in two recent merger challenges: Tronox/Cristal[1] (involving titanium dioxide), and Peabody/Arch[2] (involving coal mined in the South Powder River Basin). Private litigants have employed the homogeneous Cournot model (albeit unsuccessfully) in at least two litigated antitrust challenges.[3] Numerous antitrust matters have involved markets involving differentiated goods in which firms appear to compete by setting prices,[4] or by bargaining over price with large customers.[5] In contrast, I am not aware of an instance in which either plaintiffs or defendants argued that firms competed in quantities of differentiated goods.

Antitrust practitioners should reconsider the differentiated Cournot model. As Davis (2002) observes, while practitioners seem to default to Bertrand models when they believe products to be differentiated,and Cournot models when they believe products to be homogeneous,this default choice seems to be based on largely technical motivations and analytical tractability, and not necessarily information about the underlying mode of competition. As Eaton and Lipsey (1989) note, "any set of commodities closely related in consumption and/or in production may be regarded as differentiated products." A review of antitrust authority documents suggests that customers rarely – if ever – regard even quite similar products made by different producers as literally interchangeable, due to actual or

[1]See FTC v. Tronox Limited et al., Case No. 1:18-cv-01622 (D.D.C 2018).

[2]See FTC v. Peabody Energy Corp., Case No. 4:20-cv-00317-SEP (E.D.Mo 2020).

[3]See Concord Boat Corp. v. Brunswick Corp., 207 F.3d 1039 (8th Cir. 2000) and Heary Bros. Lightning Prot. Co. v. Lightning Prot. Institute, 287 F. Supp. 2d 1038 (D. Ariz. 2003).

[4]See FTC v. Whole Foods Market, Inc., 502 F. Supp.2d 1, 39 (D.D.C.2007) (describing how supermarkets, organic and otherwise, compete through differentiation and prices).

[5]See Hanner et al. (2016) (describing a bid model used by the FTC when litigating the Sysco/US Foods merger); FTC v. Wilhelmsen and Drew, Civil Action No. 18-cv-00414-TSC, 44 (D.D.C. 2018) (describing a merger simulation model used by the FTC's expert).

perceived differences in product quality, product type, location, or service.[6]

The theoretical literature offers additional support for the differentiated Cournot model. Salant (1986) argues that in many industries firms commit to a choice of quantity before price is set, e.g. due to lags in production, with prices set after production decisions are sunk so as to equate supply and demand. Kreps and Scheinkman (1983) argue that even when firms do appear to set prices in accordance with the Bertrand model, if the same firms first commit to quantities then outcomes match those of a Cournot model.[7] Shapiro (1989) contains an extended discussion of types of industries to which the Bertrand and Cournot models may apply. The differentiated Cournot model is most applicable to industries in which firms make strategic and lumpy decisions as to how much to produce, with prices being set later so as to equalize supply and demand.Examples of such industries may include semiconductor products, advertising, or commodity products like cement, oil, or chemicals.

"Standard" homogeneous Cournot models impose a relatively rigid form of competition that implies at least two potentially problematic results. First, homogeneous Cournot models imply that share is proportional to margin, meaning that if firm A has twice the share of firm B, firm A's price-cost margin cost must be twice that of firm B's. Defendants in Tronox/Cristal and Arch/Peabody attacked the FTC's modeling because in their view accounting margins did not match this pattern.[8] In a non-merger matter in private litigation, the court excluded the plaintiffs' Cournot model, and consequently their expert's entire report, on the basis of plaintiffs' use of a Cournot model that assumed firms with different shares had identical marginal costs.[9] In contrast, the differentiated Cournot model is consistent with a variety of shares and margins. As Werden (2010) points out, the "key test of a model used to predict the likely unilateral price effects of a merger is how well the model explains premerger pricing." The differnetiated version of the Cournot model can flexibly match pre-merger margins, while the "standard" homogeneous Cournot model cannot.

As has been pointed out by authors including Salant, Switzer, and Reynolds (1983) and Perry and Porter (1985), homogeneous Cournot models generally predict mergers to be unprofitable unless

[6]See the FTC's complaint in Tronox (*supra* note 1) at 12, describing the perceived "inferior quality" of titanium dioxide imported from China; the FTC's complaint in Peabody/Arch (*supra* note 2) discussing the unique "characteristics" and "quality" of coal from the Southern Powder River Basin.

[7]While the analysis of Kreps and Scheinkman (1983) considered homogeneous products, as Eaton and Lipsey note the analysis clearly applies with equal force to differentiated products.

[8]See Peabody Opinion, *supra* note 2, at 62, "Defendants object to Dr. Hill's applications of the Cournot model on a number of grounds, [including] that his model's predicted margins do not match observed margins."; Tronox Opinion, *supra* note 1, at 33, "(Defendants] contend[] that [...] use of the Cournot model is not appropriate and leads to results that are inconsistent with market realities.[...] Chemours' marginal cost of producing TiO2 is, according to the model, "more than [redacted] lower than the "actual" marginal cost as measured by Dr. Hill."

[9]See Heary Bros. opinion, *supra* note 3.

merging firms have sufficiently convex costs. In litigation, defendants in the Tronox/Cristal matter attacked the perceived unprofitability of the merger under the FTC's Cournot as deligitimizing the entire model.[10] For this reason, antitrust practitioners employing the homogeneous Cournot model often assume quadratic costs. In contrast, constant marginal costs present no particular difficulty under differentiated Cournot competition, and mergers in the differentiated Cournot model are more likely to be profitable than mergers involving homogeneous Cournot competitors. While there are important applications in which firms may be best conceptualized as having increasing marginal costs, researchers and practitioners may have little insight into the cost structure of a particular firm. Hence, an assumption of constant marginal cost – essentially, that firms can replicate their production process in expanding production for marginal units – is often attractive, but is all but precluded under the homogeneous Cournot model.

While merger efficiencies are a topic of considerable interest to policymakers,[11] both the theoretical and empirical literature on merger efficiencies is thin. Notable exceptions include Werden (1996) and Froeb and Werden (1998), which derive comparable $CMCR$ metrics for differentiated Bertrand competition and homogeneous Cournot competition, respectively. The Werden (1996) and Froeb and Werden (1998) $CMCR$ metrics are commonly used by antitrust practitioners,[12] both because of their direct relevance to antitrust questions and because they require few assumptions, e.g. on demand and cost curvature. My paper establishes a comparable $CMCR$ metric for settings in which competition is differentiated in quantities.

Section 2 develops a numerical example illustrating key differences between Bertrand and Cournot competition, and demonstrating the application of $CMCR$. Section 3 derives the differentiated Cournot $CMCR$. Section 4 discusses calibration of differentiated Cournot demand systems and merger simulation. Section 5 applies differentiated Cournot modeling to a hypothetical industry, and compares the differentiated Cournot model to the "standard" Cournot and Bertrand models. Section 6 concludes.

2 Numerical example

This section contains a numerical example that demonstrates the distinction between Bertrand and Cournot modes of competition among producers of differentiated products, the application of $CMCR$, and how $CMCR$ varies by mode of competition. While both Bertrand and Cournot $CMCR$ are invariant to demand and cost curvature, it is helpful to work with particular functional forms to easily allow calculation of merger price effects. In particular, throughout this section assume that two

[10] See Tronox Opinion, *supra* note 1, at 33.

[11] See Wilson (2020).

[12] See Greenfield et al. (2019) for a discussion of $CMCR$ as used in the Tronox/Cristal litigation.

firms produce differentiated products at constant marginal cost c_i, with demand for products 1 and 2 given by:

$$\begin{aligned} q_1 &= 10 - p_1 + \tfrac{1}{2}p_2 \\ q_2 &= 10 - p_2 + \tfrac{1}{2}p_1 \end{aligned} \tag{1}$$

Bertrand

The Bertrand solution follows directly from the two firms' first-order conditions for profit maximization in p_i:

$$\begin{aligned} p_1 &= \tfrac{20}{3} + \tfrac{8}{15}c_1 + \tfrac{2}{15}c_2 \qquad & p_2 &= \tfrac{20}{3} + \tfrac{8}{15}c_2 + \tfrac{2}{15}c_1 \\ q_1 &= \tfrac{20}{3} - \tfrac{7}{15}c_1 + \tfrac{2}{15}c_2 \qquad & q_2 &= \tfrac{20}{3} - \tfrac{7}{15}c_2 + \tfrac{2}{15}c_1 \end{aligned}$$

Cournot

The Cournot solution, in which firms choose quantities, can be calculated in one of two equivalent ways. First, the demand system (1) can be inverted, and thus expressed with prices as the dependent variables. Using $\mathbf{p}$ and $\mathbf{q}$ to represent 2×1 price and quantity vectors, $\mathbf{A}$ to represent a 2×1 vector of intercepts, and $\mathbf{B}$ to denote a 2×2 matrix of coefficients, we can rewrite system (1) as $\mathbf{q} = \mathbf{A} - \mathbf{B}\mathbf{p}$. It then follows that $\mathbf{p} = \mathbf{B}^{-1}\mathbf{A} - \mathbf{B}^{-1}\mathbf{q}$, or:

$$\begin{aligned} p_1 &= 20 - \tfrac{4}{3}q_1 - \tfrac{2}{3}q_2 \\ p_2 &= 20 - \tfrac{4}{3}q_2 + \tfrac{2}{3}q_1 \end{aligned} \tag{2}$$

Given system (2), the Cournot solution is easily obtained from the two firms' first-order conditions for profit maximization in q_i, and is given below in expression (3).

$$\begin{aligned} p_1 &= 8 + \tfrac{7}{15}c_1 + \tfrac{2}{15}c_2 \qquad & p_2 &= 8 + \tfrac{7}{15}c_2 + \tfrac{2}{15}c_1 \\ q_1 &= 6 - \tfrac{2}{5}c_1 + \tfrac{1}{10}c_2 \qquad & q_2 &= 6 - \tfrac{2}{5}c_2 + \tfrac{1}{10}c_1 \end{aligned} \tag{3}$$

An alternative method for obtaining the Cournot solution, described in Jaffe and Weyl (2013), is less direct, but perhaps more illustrative of the difference between the Bertrand and Cournot modes of competition. While Cournot competition is traditionally defined by firms choosing quantity and Bertrand competition by firms choosing price, this is not the essential distinction between Bertrand and Cournot competition. In fact, given the one-to-one mapping between price and quantity defined by a firm's demand curve the distinction is entirely artificial. Instead, what distinguishes the two modes of competition is how firms expect their rivals to react to a change in either price or quantity. Under Bertrand competition, a firm contemplating increasing its own price reasons that its rivals will hold price steady in response but will let quantity adjust as needed. Under Cournot competition, such a firm reasons that its rivals will increase price in response, so as to maintain their quantities at a constant level.

It follows that the Cournot solution can be obtained by allowing firms to optimize in prices, but with conjectures that rivals' prices will adjust to own price changes so as to hold steady rivals' quantities. In the two-firm oligopoly described by system (1), firm i chooses p_i so as to maximize profits, given a belief that firm j will adjust p_j so that $q_j(p_j(p_i), p_i) = K_j$, where K_j is firm j's constant quantity. Then, firm i's profit-maximization problem is:

$$\max_{p_i}(10 - p_i + \frac{1}{2}p_j)(p_i - c_i) \quad \text{subject to:} \quad K_j = 10 - p_j + \frac{1}{2}p_i \tag{4}$$

Solving the constraint for p_j, inserting it into the objective function, and solving produces two first-order conditions: $p_i = 10 - \frac{1}{3}K_j$. Combining these first-order conditions with the two constraints yield a system of four linear equations in four unknowns. It is direct that the solution to this system is given by equation (3) above.

Monopoly

Following a hypothetical merger of firms 1 and 2, the monopoly solution does not depend on mode of competition. This is because the distinction between Bertrand and Cournot – how rivals are thought to react to a change in price – collapses under monopoly. Using standard techniques, the monopoly solution is:

$$\begin{array}{cc} p_1 = 10 + \frac{1}{2}c_1 & p_2 = 10 + \frac{1}{2}c_2 \\ q_1 = 5 - \frac{1}{2}c_1 + \frac{1}{4}c_2 & q_2 = 5 - \frac{1}{2}c_2 + \frac{1}{4}c_1 \end{array} \tag{5}$$

Constant marginal cost reduction ($CMCR$)

$CMCR$ gives the reduction in costs necessary to maintain pre-merger pricing following a merger. $CMCR$ is independent of demand curvature, and thus applies to the linear demand example here. To demonstrate the use of $CMCR$, suppose $c_i = 10$, $i = 1, 2$, so that symmetric equilibria obtain. Then, the Bertrand duopoly price is $\frac{40}{3}$, the Cournot duopoly price is 14, and the monopoly price is 15.

From equation (5) of Werden (1996), the Bertrand $CMCR$ is $\frac{1}{3}$, meaning that if a merger results in efficiencies that lower each merging firm's costs by at least $\frac{1}{3}$, the monopolist's prices will be no greater than the duopolists prices.[13] As expected, we see from expression (5) that were costs to decrease by $\frac{1}{3}$ following the merger, to $c_i = \frac{20}{3}$, the monopolist would set prices of $p_i = \frac{40}{3}$, equal to the Bertrand duopoly prices.

From proposition 1 in section 3 of this paper, below, the Cournot $CMCR$ is $.2$. From expression (5), were costs to decrease by 20% following the merger, to $c_i = 8$, the monopolist would set prices $p_i = 14$, identical to prices under Cournot competition. That the Cournot $CMCR$ is lower than the Bertrand $CMCR$ is due to the fact that Cournot duopoly prices are greater than Bertrand duopoly

[13]To compute the Bertrand $CMCR$, note that margins are $m_i = .25$ and diversion ratios are $D_{ij} = .5$.

Finally, while this example has sufficient detail to clearly illustrate the mechanics of $CMCR$, as a general matter $CMCR$ requires neither information about demand curvature nor knowledge of post-merger equilibrium. Instead, $CMCR$ requires only 1- knowledge of pre-merger prices, margins, and diversion, and 2- a view as to mode of competiton (Bertrand or Cournot).

3 $CMCR$ under differentiated Cournot competition

N separately-owned single-product firms produce differentiated goods. The firms compete by simultaneously choosing quantities, with each firm then receiving the market clearing price for its quantity. Each firm's price depends on both its own quantity and its rivals' quantities, according to the inverse demand curve $p_i(Q)$, where Q is a vector with generic element q_i. Assume that $p_i(Q)$ is differentiable, with $\frac{\partial p_i}{\partial q_j} < 0$ for all i, j, so that products produced by the N firms are substitutes. Firm i's cost curve is given by $c_i(q_i)$, with $c_i(q_i)$ differentiable and with $c_i' > 0$. Let $m_i = \frac{p_i - c_i'(q_i)}{p_i}$ denote firm i's margin over its cost on its last unit sold.

Firm i chooses q_i so as to maximize its profits. Firm i's first-order condition for profit-maximization is:

$$m_i^{pre} = -\frac{q_i}{p_i}\frac{\partial p_i}{\partial q_i} \tag{6}$$

Now suppose that firms 1 and 2 merge. Each merging firm now chooses q_i to maximize the sum of firm 1's and firm 2's profits. Post-merger, firms 1 and 2 have the following first-order conditions:

$$m_i^{post} + \frac{\partial p_j}{\partial q_i}\frac{q_j}{p_i} = -\frac{q_i}{p_i}\frac{\partial p_i}{\partial q_i} \text{ for } i = 1, 2 \tag{7}$$

where m_i^{post} denotes post-merger margin. In general, prices, quantities, and slopes differ before and after a merger. For reasons that will become obvious, we suppress the superscript on all terms other than margin.

If the merger between firms 1 and 2 lowers one or both of these firms' marginal cost, this will raise their margin, all else equal. Proposition 1 derives the amount by which each merging firm's margin would need to increase – and consequently, the amount by which its marginal cost would need to decrease – in order for the merger not to result in a price increase. Following Werden (1996) and Froeb and Werden (1998), I refer to this quantity as firm i's compensating marginal cost reduction ($CMCR$).

Proposition 1. *Following a merger of firms 1 and 2, the amount by which each of the merging firms' marginal costs must decrease so that post-merger quantities and prices are unchanged from pre-*

merger quantities and prices is:

$$CMCR_i = \frac{\frac{\partial p_j}{\partial q_i}}{\frac{\partial p_j}{\partial q_j}} \frac{m_j \frac{p_j}{p_i}}{1 - m_i} \textit{ for } i, j \in \{1, 2\} \tag{8}$$

Proof. If the post-merger outcome matches the pre-merger outcome, it follows that all terms other than margin are the same in both equations (6) and (7), for firms 1 and 2. The proof proceeds by solving for the implied value of m_i^{post}, as a function of m_i^{pre}.

Multiply the middle term in firm i's post-merger first-order condition (7) by $1 = \frac{\frac{\partial p_j}{\partial q_j}}{\frac{\partial p_j}{\partial q_j}} \frac{p_j}{p_j}$. Then, substitute firm j's pre-merger first-order condition (6) into equation (7), to yield the following:

$$m_i^{post} = m_i^{pre} + \frac{\frac{\partial p_j}{\partial q_i}}{\frac{\partial p_j}{\partial q_j}} m_j^{pre} \frac{p_j}{p_i} \text{ for } i = 1, 2,\ j \neq i \tag{9}$$

Following equation (1) of Werden (1996), the relationship between the change in marginal cost and the change in margin is:

$$\frac{c_i^{pre} - c_i^{post}}{c_i^{pre}} = \frac{m_i^{post} - m_i^{pre}}{1 - m_i^{pre}} \tag{10}$$

Substituting the expression for m_i^{post} from (9) into (10) yields the expression for $CMCR_i$. ■

$CMCR$ depends only on pre-merger values. The ratio of slopes, $\frac{\frac{\partial p_j}{\partial q_i}}{\frac{\partial p_j}{\partial q_j}}$, measures the closeness of substitutability between goods 1 and 2. If the ratio is high, than an increase in j's quantity q_j affects i's price p_i nearly as much as does an increase in i's own quantity q_i, so i and j are close substitutes. On the other hand, if the ratio is small, than j's price is much less affected by i's quantity than by j's own quantity, and the products are more distant substitutes. The margins m_j and m_i are related to the responsiveness of the merging firms' demand to changes in price/quantity via the pre-merger first-order conditions (6). From inspection of (6), a higher margin implies that the firm's demand is less responsive to changes in price or quantity.

In some cases, the demand slopes in the expression for $CMCR$, $\frac{\partial p_j}{\partial q_i}$ and $\frac{\partial p_j}{\partial q_j}$, may be directly measurable. This is most likely if price and quantity information spanning contraction and expansion events is available. In other cases, directly estimating these demand slopes may prove difficult. The following section provides an algorithm to calibrate demand slopes using pre-merger information on margins and diversions ratios. The resulting demand slopes are sufficient both to calculate $CMCR$, and to calibrate particular parameterized demand systems to pre-merger information, in order to simulate the merger price effect.

4 Calibration and merger simulation

Researchers and antitrust practitioners often populate the parameters of a demand system using either estimation – meaning econometric identification using variation in a dataset – or calibration – meaning fitting parameters as closely as possible to a relatively small number of observed statistics, commonly margins, diversion ratios,[14] or cost pass through terms.[15] As noted by Miller et al. (2012), antitrust practitioners more commonly use calibration, as confidential information available through subpoenas and discovery commonly suffice to measure diversion ratios and margins, but lack the detail required for econometric identification.

In this section, I discuss what inferences can be made about demand slopes – meaning own and cross price derivatives – from the margins and diversions ratios that are commonly available to antitrust practitioners. From proposition 1, these demand slopes suffice to calculate $CMCR$. I focus on demand slopes, rather than assuming functional forms for demand and cost functions, to emphasize both that $CMCR$ is invariant to the curvature of the demand and cost functions and that the calibrated slopes can be fit to a variety of demand and cost curves, depending on the application. For simplicity, I focus on the case of an industry with N firms, for which all N margins and all $(N^2 - N)$ diversions are observable. This case may include industries consisting of publicly traded firms (who generally report profit margins) and for which an assumption of diversion proportional to share is appropriate.

I proceed by adapting existing calibration techniques for fitting demand curves to market observables when competition is in prices. In this setting, calibration chooses own price coefficients so that the margins implied by each firm's first-order condition closely matches observed margins, while choosing both own- and cross-price coefficients so as to match the implied diversion ratios to observed diversion ratios as closely as possible, all while satisfying Slutsky symmetry. If all relevant margins and diversion ratios are observed, calibration is generally overidentified, meaning that there is a tradeoff between more closely matching implied to observed margins, and more closely matching implied to observed diversion ratios. This tradeoff arises because under Slutsky symmetry any vector of own price coefficients implies multiple values of each cross-price coefficient; not only must a calibrated cross-price coefficient match both of these values as closely as possible, but the calibrated own-price coefficients affect the fit of implied to actual diversions, in addition to implying margins.

In setting of this paper, where competition is in quantities, calibration involves the same tradeoff between margins and diversion ratios. Mode of competition determines each firm's first-order condition, and thus the margin implied by a calibrated matrix of demand slopes. However, diversion ratios

[14]The diversion ratio from firm i to firm j is commonly defined by $D_{ij} = -\frac{\frac{\partial q_j}{\partial p_i}}{\frac{\partial q_i}{\partial p_i}}$, and represents the share of firm i's marginal customers that would switch to firm j were firm i no longer an option.

[15]For a discussion of using observations on cost pass-through to calibrate demand, see Miller et al. (2012).

are a property of the underlying demand system itself, and do not depend on whether competition is in prices or quantities. Because diversion ratios are generally defined with respect to the direct demand system $Q(P)$, I calibrate both a matrix of direct demand slopes $\frac{\partial Q}{\partial P}$ and the corresponding matrix of indirect demand slopes, $\frac{\partial P}{\partial Q} = \left(\frac{\partial Q}{\partial P}\right)^{-1}$.[16]

Before describing the calibration algorithm, one addition to the model is needed. I have described the N firms as a "market." In any market, when customers switch from one firm, some customers go to other firms in the market, a presumably smaller number of customers switch to firms out of the market, and some customers may simply purchase less of the good in question. To capture this, I assume out-of-market diversion equal to $z \geq 0$. Depending on the application, z may be measured econometrically, or may be proxied for. For example, in some matters comparatively little is known about customer switching to manufacturers in China; in this case, Chinese manufacturers could be classified as "out-of-market", with out-of-market diversion proportional to a measure of the combined shares of Chinese manufacturers.

4.1 Calibration of demand slopes

My calibration algorithm first chooses a vector of own price coefficients to populate the diagonal of $\frac{\partial Q}{\partial P}$, the matrix of direct demand slopes. It then chooses off-diagonal elements of $\frac{\partial Q}{\partial P}$ to match implied diversions to observed diversion as closely as possible under Slutsky symmetry. It then inverts $\frac{\partial Q}{\partial P}$ to obtain $\frac{\partial P}{\partial Q}$, the matrix of inverse demand slopes. Then, it calculates an error function increasing in: 1- the squared distance between the diversion ratios as implied by $\frac{\partial Q}{\partial P}$ and observed diversion ratios, and 2- the squared distance between the margins implied by profit maximization and $\frac{\partial P}{\partial Q}$ and observed margins. Finally, the algorithm iterates over different choices of own price coefficients until the error function is numerically minimized.

First, I fix notation. Let B denote an $N \times N$ matrix of calibrated price slopes with generic element b_{ij}. To economize notation, all elements of B are positive, so that $b_{ii} = -\frac{\partial q_i}{\partial p_i}$ and $b_{ij} = \frac{\partial q_i}{\partial p_j}$. Let β denote the corresponding matrix of calibrated quantity slopes, so that $\beta = \overline{B}^{-1} = \frac{\partial Q}{\partial P}$, where $\overline{B}$ is equal to B but with its diagonal elements replaced by $-b_{ii}$, so that $\overline{B} = \frac{\partial Q}{\partial P}$. Let the generic elements of B and β be denoted by b_{ij} and β_{ij} respectively.

Steps 1-6 below construct an error function for any choice of own price coefficients b_{ii}. Calibration of demand slopes then proceeds by choosing $\{b_{ii}\}_{i=1}^{N}$ so as to minimize this error function.

1. For a given choice of $\{b_{ii}\}_{i=1}^{N}$, Slutsky symmetry and the definition of diversion (from footnote

[16] Slutsky symmetry implies that both $\frac{\partial P}{\partial Q}$ and $\frac{\partial Q}{\partial P}$ are symmetric matrices.

14) imply:

$$b_{ij} = b_{ji} = D_{ij} b_{ii} = D_{ji} b_{jj} \tag{11}$$

It is not generally possible for equation (11) to hold for all i, j. Indeed, a choice of b_{ii} implies that $b_{ij} = b_{ii} D_{ij}$, while a choice of b_{jj} implies $b_{ij} = b_{ji} = b_{jj} D_{ji}$. Calculate b_{ij} to be the weighted average of these two terms, with weights given by relative shares, so that:

$$\hat{b}_{ij} = \hat{b}_{ji} = \left(\frac{s_i}{s_i + s_j} b_{ii} D_{ij} + \frac{s_j}{s_i + s_j} b_{jj} D_{ji} \right) \tag{12}$$

2. A choice of $\{b_{ii}\}$ and $\hat{b}_{ij}$ imply a set of diversion ratios, $\hat{D}_{ij} = \frac{\hat{b}_{ji}}{b_{ii}}$. Scale the off-diagonal coefficients $\hat{b}_{ij}$ so that $\min(\sum_{j \neq 1} D_{1j}, \sum_{j \neq 2} D_{2j}) \leq (1-z)$. That is, $b_{ij} = \hat{b}_{ij} * \min \left\{ \frac{b_{11}(1-z)}{\sum_{j \neq 1} \hat{b}_{j1}}, \frac{b_{22}(1-z)}{\sum_{j \neq 2} \hat{b}_{j2}}, 1 \right\}$ for all $i, j, i \neq j$.[17]

3. Using the initial choice of $\{b_{ii}\}_{i=1}^{N}$ and the values of $\{b_{ii}\}_{i \neq j}$ implied by step 2, construct $\overline{B}$ whose diagonal elements equal $-b_{ii}$ and whose off-diagonal elements equal b_{ij}. Then, define $\beta = \overline{B}^{-1}$.

4. For each firm i, a choice of b_{ii} implies a margin of $\tilde{m}_i = \beta_{ii} \frac{q_i}{p_i}$ for firm i. Assign weight $\pi * w_i$ to the squared difference between implied margin $\tilde{m}_i$ and the observed margin m_i, with $\sum_{i=1}^{N} w_i = 1$.

5. The choices of $\{b_{ii}\}$ and $\{b_{ij}\}$ imply values of diversion $\tilde{D}_{ij} = \frac{b_{ji}}{b_{ii}}$. The implied $\tilde{D}_{ij}$ generate $N * (N-1)$ error terms between implied and observed diversion. Assign weight $(1-\pi)\omega_{ij}$ to the squared difference between the implied $\tilde{D}_{ij}$ and the observed D_{ij}, with $\sum_{i,j} \omega_{ij} = 1$.

   The choices of w_i and ω_{ij} I use in my calibration are:

$$w_i = s_i \tag{13}$$

$$\omega_{ij} = \frac{s_i s_j}{1 - \sum_{i=1}^{N} s_i^2} \tag{14}$$

   It is straightforward that $\sum_{i=1}^{N} w_i = \sum_{i=1}^{N} \sum_{j \neq i} w_{ij} = 1$, as required. I have not found that calibrated demand slopes to vary greatly in π, but in the calibrations discussed in section 5, I set $\pi = .99$.

[17] Other choices of scalars for off-diagonal terms are defensible; the one given in the text is the most conservative, in that it results in the lowest diversion ratios. An alternative is an average of the $\frac{b_{ii}(1-z)}{\sum_{j \neq i} \hat{b}_{ji}}$ terms over $i = 1, 2$.

6. Define the error function $\xi(\{b_{ii}\}_{i=1}^{N})$ as follows:

$$\xi(\{b_{ii}\}_{i=1}^{N}) = \pi \sum_{i=1}^{N} w_i(\tilde{m}_i - m_i)^2 + (1-\pi) \sum_{i=1}^{N} \sum_{j \neq i} \omega_{ij}(\tilde{D}_{ij} - D_{ij})^2 \tag{15}$$

Given steps 1-6, calibration of demand slopes reduces to:

$$\min_{\{b_{ii}\}_{i=1}^{N}} \xi(\{b_{ii}\}_{i=1}^{N}) \tag{16}$$

In practice, I solve equation (16) numerically.[18]. Given a solution to equation (16) consisting of $\{b_{ii}\}_{i=1}^{N}$, define the matrix B via steps 1-2 above, and the matrix β by step 3.

Corollary 2 incorporates the calibrated matrix β into $CMCR$ as defined in proposition 1. Its proof is immediate.

Corollary 2. *Given a set of calibrated demand derivatives B and β, the amount by which each of the merging firms' marginal costs must decrease so that post-merger quantities and prices are unchanged from pre-merger quantities and prices is following a merger of firms 1 and 2 is:*

$$CMCR_i = \frac{\beta_{ji}}{\beta_{jj}} \frac{m_j \frac{p_j}{p_i}}{1 - m_i} \textit{ for } i, j \in \{1, 2\} \tag{17}$$

4.2 Merger simulation

Industrial organization researchers and antitrust practitioners often wish to calculate the counterfactual effect of some event, such as a merger. Doing so generally requires a fully-specified demand system and set of cost curves, and the counterfactual effect will generally depend on assumptions made about the form and curvature of these functions (unlike $CMCR$, which is invariant to the curvature of the underlying demand system and cost curves). Hence, this section moves beyond the demand-agnostic calibration of slopes discussed in the previous section to additionally calibrate demand intercept terms.

Merger simulation as applied by researchers and antitrust practitioners involves resolving profit maximization problems for the merging firms given calibrated or estimated demand and cost curves, and determining the implied merger price effect. While merger simulation requires much stronger assumptions than does $CMCR$, it also produces more granular, and thus potentially more useful, information. Specifically, while $CMCR$ gives the cost reduction for each merging firm that would result in no price increase, it it silent on the amount of any price change should expected merger

[18] Matlab code solves the minimization problem in equation (16), given user-inputted values for margins, diversions, out-of-market diversion, and weight π

efficiencies not precisely equal $CMCR$. Second, $CMCR$ can produce an ambiguous answer when applied to a specific merger; suppose that $CMCR_1 = 7\%$, while $CMCR_2 = 3\%$, and both merging firms are expected to reduce their marginal costs by 5%. In this case, $CMCR$ is insufficient to evaluate even the sign of the merger's total effect on price.

I discuss parameterizing both linear (section 4.2.1) and loglinear (section 4.2.2) demand curves from the matrix of demand slopes β calibrated from pre-merger observables in section 4.1. Section 4.2.3 discusses parameterizations of other demand systems.

4.2.1 Linear demand

First, assume that demand is linear, so that the derivative matrices B and β describe the slope of demand for any values of P and Q, not just those prevailing pre-merger. While merger simulation admits a myriad of potential assumptions on cost curves, for simplicity assume that each firm's cost curve is either linear ($c_i(q_i) = c_i * q_i$) or quadratic ($c_i(q_i) = \frac{\gamma_i}{2} q_i^2$, with γ_i calibrated to match firm i's observed pre-merger margin m_i).

All firms' pre-merger first-order condition are given by equation (6) in section 3. Determine calibrated matrix β using the algorithm in section 4.1. Then, should firms 1 and 2 merge these firms would have the following first-order conditions:

$$\alpha_i - 2\beta_{ii}q_i - \gamma_i q_i - 2\beta_{ij}q_j - \sum_{k=3}^{N} \beta_{ik}q_k = 0 \;\; i, j \in \{1, 2\} \text{ (if firm } i \text{ has quadratic cost)}$$

$$\alpha_i - 2\beta_{ii}q_i - c_i - 2\beta_{ij}q_j - \sum_{k=3}^{N} \beta_{ik}q_k = 0 \;\; i, j \in \{1, 2\} \text{ (if firm } i \text{ has constant marginal cost)}$$

The non-merging firms have the following first-order conditions:

$$\alpha_i - 2\beta_{ii}q_i - c_i - \sum_{k \neq i}^{N} \beta_{ik}q_k = 0 \;\; i, j \in \{1, 2\} \text{ (if firm } i \text{ has constant marginal cost)}$$

$$\alpha_i - 2\beta_{ii}q_i - \gamma_i q_i - \sum_{k \neq i}^{N} \beta_{ik}q_k = 0 \;\; i, j \in \{1, 2\} \text{ (if firm } i \text{ has quadratic cost)}$$

If all firms have quadratic costs, the conditions for firms $3, ..., N$ are adjusted accordingly.

Merger simulation solves the system of first-order conditions, and compares the resulting prices and quantities to observed pre-merger prices and quantities.[19]

[19] I first compute pre-merger equilibrium prices and quantities implied by calibrated demand, which may slightly differ from observed prices and quantities. I then use these implied pre-merger equilibrium values as the baseline, so that any imprecision in calibration applies equally to pre- and post-merger prices and quantities, isolating the effect of the merger.

4.2.2 Loglinear demand

Now assume that demand is loglinear, so that the direct demand curve is given by:

$$\log(q_i) = \gamma_i + \sum_{j=1}^{N} \epsilon_{ij} \log(p_j) \tag{18}$$

where $\epsilon_{ij} = \frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i}$ is the own elasticity of demand.

Equation (18) can be inverted to produce inverse demand, as follows:

$$\log(p_i) = \eta_i - \sum_{j=1}^{N} \sigma_{ij} \log(q_j) \tag{19}$$

It is direct that $\sigma_{ij} = \frac{\partial \log(p_i)}{\partial \log(q_j)} = \frac{\partial p_i}{\partial q_j}\frac{q_j}{p_i}$. Under the calibration discussed in section 4.1, $\frac{\partial p_i}{\partial q_j} = \beta_{ij}$. Therefore, the vector η and the matrix σ are calibrated from demand slopes β as follows:

$$\sigma_{ij} = \beta_{ij}\frac{q_j}{p_i} \tag{20}$$

$$\eta = \log(P) + \sigma * \log(Q) \tag{21}$$

Under separate ownership, each of N firms simultaneously chooses quantity q_i in order to maximize profits given rivals' quantity choices. A pre-merger Nash equilibrium satisfies:

$$\begin{aligned} & p_i = \frac{MC_i}{1-\sigma_i} \text{ for } i = 1, ..., N \\ \Rightarrow & \sum_{j=1}^{N} \sigma_{ij} \log(q_j) = \eta_i - \log(MC_i) + \log(1 - \sigma_i^c) \text{ for } i = 1, ..., N \\ \Rightarrow & Q = e^{\sigma^{-1}K} \end{aligned} \tag{22}$$

Equation (22) provides a closed form solution for the pre-merger equilibrium in quantities. By design, the quantities implied by equation (22) closely match observed quantities.

Following a merger of firms 1 and 2, each merging firm internalizes the effect of its quantity choice on the profits of the other. The resulting post-merger first order conditions are:

$$e^{\theta_1}(1-\sigma_{11}) - e^{\theta_2}\sigma_{21}\frac{q_2}{q_1} - c_1 = 0 \tag{23}$$

$$e^{\theta_2}(1-\sigma_{22}) - e^{\theta_1}\sigma_{12}\frac{q_1}{q_2} - c_2 = 0 \tag{24}$$

$$e^{\theta_j}(1-\sigma_{jj}) - c_j = 0 \text{ for } j = 3, ..., N \tag{25}$$

where $\theta_i = \gamma_i - \sum_{j=1}^{N} \sigma_{ij} \log(q_j)$, so that $e^{\theta_i} = p_i$ equals firm i's price, as a function of logged quantities. This system of first-order conditions in (23)-(25) is solved numerically for q_i, $i = 1, ..., N$. As with linear demand, the solution to system (23)-(25) is compared to the pre-merger solution (22), with the difference (both in quantities and in associated prices) taken to be the effect of the merger.

4.2.3 Other demand systems

The algorithm I describe in section 4 calibrates the slopes of both the direct and inverse demand curves at the pre-merger equilibria. Various assumptions on direct demand curves are used by researchers and practitioners modeling competition in prices, with merger simulation results depending on the assumed form of demand. I chose the linear and loglinear systems as exemplars because of their relative tractability, and because the linear model in particular is commonly used in the industrial organization literature and by antitrust practitioners.[20] However, any direct demand system that is invertible in a large enough neighborhood of the pre-merger equilibrium and which can be calibrated from direct and/or inverse demand slopes can be used to model competition in quantities with differentiated goods. Okuguchi (1987) discusses assumptions on the direct demand system necessary for it to be invertible, and for comparisons between Bertrand and Cournot outcomes to be well-founded.

5 Applications

In this section, I consider a hypothetical industry – consisting of five differentiated firms with "observed" margins and shares given in table 1 – and discuss applying the differentiated Cournot version of $CMCR$, calibrating demand slopes, and merger simulation. I do so through a series of brief examples that illustrate the relative advantages of the differentiated Cournot model over the "standard" homogeneous Cournot model – including the ability to flexibly match observed margins that are not proportionate to shares, and the relative profitability of differentiated Cournot mergers. I also contrast the differentiated Cournot model to the oft-used Bertrand and homogeneous Cournout models to situations where differentiated firms compete in quantities. For the sake of tractability, I assume that diversion is proportion to share, with 10% out of market diversion, and I use weighting parameter $\pi = .99$ in calibrating demand slopes.

5.1 Flexibility in matching observed margins

As discussed in section 1, the "standard" homogeneous Cournot model predicts that each firm's market share is linearly related to its profit margin, via its first order condition $m_i = -\frac{s_i}{\epsilon_{ii}}$, where s_i is firm i's share, and ϵ_{ii} its own price elasticity of demand. In practice, observed margins rarely obey the homogeneous Cournot model's prescription, and this incongruence is sometimes used to attack the validity of the Cournot model.[21] Unlike the homogeneous Cournot model, the differentiated Cournot

[20] Papers modeling competition in quantities with differentiated goods include: Alipranti, Millou, and Petrakis (2014); Davis (2002), Häckner (2000), Qiu (1997), Singh and Vives (1984), Okuguchi (1987), and Vives (1984).

[21] See footnotes 8 and 9, *supra*, and surrounding discussion.

Firm	Market share	Margin
1	24.0%	32.0%
2	18.0%	13.0%
3	36.0%	30.0%
4	16.0%	27.0%
5	6.0%	20.0%

Table 1: Observed margins and market shares for a hypothetical industry, used throughout section 5.

model is consistent with *any* relationship between share and margin, as it allows the various firms to have distinct own-price elasticities.

Given the shares and margins in table 1, I calibrate demand slopes using the algorithm described in section 4.1 to be:

$$\beta = \frac{\partial P}{\partial Q} = \begin{bmatrix} 0.0133 & 0.0046 & 0.0059 & 0.0068 & 0.0070 \\ 0.0046 & 0.0073 & 0.0040 & 0.0047 & 0.0050 \\ 0.0059 & 0.0040 & 0.0083 & 0.0061 & 0.0063 \\ 0.0068 & 0.0047 & 0.0061 & 0.0169 & 0.0071 \\ 0.0070 & 0.0050 & 0.0063 & 0.0071 & 0.0332 \end{bmatrix} \tag{26}$$

I calculate the margins implied by the demand slopes, $\tilde{m}_i = \beta_{ii}\frac{q_i}{p_i}$, using step 4 of section 4.1. Table 2 displayes these implied margins, along with "observed" margins from table 1. The rightmost column of table 2 displays margins implied by alternatively calibrating the observables in table 1 to a homogeneous demand system.[22] By inspection of table 2, while differentiated Cournot calibration can flexibly match observed margins, homogeneous Cournot calibration implies margins that are quite different from those observed. The reason is that homogeneous Cournot calibration chooses one parameter – market elasticity of demand – to match a weighted average of observed margins, whereas differentiated Cournot calibration allows each firm to have different own elasticities.

[22]To calibrate a market demand curve under the assumption of homogeneous Cournot competition, I calibrate the market elasticity of demand ϵ that most closely matches the observed margins and shares from table 1. Specifically, I choose the value of elasticity ϵ that minimizes the sum of squared errors between implied and observed margins, weighted by share, or:

$$\epsilon^* = argmin_\epsilon \sum_{i=1}^{N} s_i \left(\frac{s_i}{\epsilon} - m_i\right)^2 \tag{27}$$

Then, each firm's implied margin – as reported in table 2 – is $m_i = \frac{s_i}{\epsilon^*}$.

	Observables		Implied margins	
Firm	Market share	Margin	Diff. Cournot	Homog. Cournot
1	24.0%	32.0%	31.9%	23.5%
2	18.0%	13.0%	13.2%	17.6%
3	36.0%	30.0%	30.0%	35.3%
4	16.0%	27.0%	27.0%	15.7%
5	6.0%	20.0%	19.9%	5.9%

Table 2: Observed margins and those implied by calibrating observables to a differentiated Cournot model and a homogeneous Cournot model, respectively.

5.2 Merger profitability

Section 1 discussed literature documenting the apparent unprofitability of mergers implied by the "standard" homogeneous Cournot model, as well as litigated matters in whcih the purported unprofitability of Cournot mergers undermined the credibility of the homogeneous Cournot model.[23] Because the products are more distant substitutes in the differentiated Cournot model, a reduction in the merging firms' output creates less of an incentive for nonmerging firms to expand production than would be the case were the various firms' products perfect substitutes. Because the expansion of output by nonmerging firms necessarily lowers the profit of the merging firms, differentiated Cournot mergers are thus relatively more profitable than homogeneous Cournot mergers.

To illustrate the relative profitability of a merger of differentiated Cournot competitors, I first fit linear demand curves to the demand slopes described by equation (26). Then, I fit a market demand curve to the homogeneous Cournot market elasticity described in footnote 22. I apply section 4.2.1 to calculate the merger price effects from a merger of firms 1 and 2 under differentiated Cournot, with a corresponding calculation for homogeneous Cournot. Finally, I calculate the profitability of the merger to the merging firms, $\pi_{1,2}^{post} - (\pi_1^{pre} + \pi_2^{pre})$, for both differentiated and homogeneous Cournot.

Results are shown in table 3, for each of three different assumptions on the curvature of cost curves (all firms have linear costs, merging firms have quadratic costs while nonmerging firms have linear costs, and all firms have quadratic costs).[24] From table 3, a merger under a homogeneous Cournot demand system is considerably more unprofitable than a merger under a differentiated Cournot demand

[23] See footnote 10, and surrounding text.

[24] Linear cost curves have form $c_i(q) = k_i q_i$, quadratic curves have form $c_i(q_i) = \frac{\gamma_i}{2} q_i^2$. I omit the case of homogeneous Cournot competitors with linear costs from table 3, because the homogeneous Cournot model does not admit a post-merger equilibrium in which both merging firms continue to exist (see Salant, Switzer, and Reynolds, 1983).

	Differentiated Cournot			Homogeneous Cournot	
	Linear demand			Linear demand	
Firm	Linear costs	Merging quadratic	Quadratic costs	Merging quadratic	Quadratic costs
1	3.1%	1.8%	2.1%	1.6%	3.7%
2	4.9%	1.3%	1.5%	1.6%	3.7%
3	1.5%	0.6%	1.0%	1.6%	3.7%
4	1.5%	0.6%	1.1%	1.6%	3.7%
5	1.5%	0.6%	1.1%	1.6%	3.7%
	-1.8%	-0.5%	0.0%	-4.7%	-0.8%
	Profitability of merger to merging firms				

Table 3: Simulated price increases (blue rows) and merger profitability (peach rows) following a merger of firms 1 and 2, for demand calibrated to a differentiated Cournot linear demand system or a homogeneous Cournot linear demand system.

system. It follows that a lower magnitude of merger efficiencies would "rationalize" the merger under a differentiated Cournot competition.

5.3 Efficiencies

Application of corollary 2 to the calibrated demand slopes in equation (26) provides values of $CMCR$ under differentiated Cournot. Table 4 reports these, and corresponding values of $CMCR$ obtained by calibrating a homogeneous Cournot demand system using equation (27) and applying Froeb and Werden (1998). In this section's example, much lower efficiencies are needed to offset merger price effects when products are differentiated.

Next, I reconsider the merger simulations of section 5.2 for various potential values of merger cost savings realized by the two merging firms, with an additional case of loglinear demand. Table 5 lists price increases resulting from a merger of firms 1 and 2 for cost savings of 2%, 6%, and the values of $CMCR$ for both differentiated and homogeneous Cournot demand from table 4. The table also lists the profitability of the merger to the merging firms under each scenario.

While modest cost savings below $CMCR$ render a merger profitable under differentiated Cournot demand, considerably greater costs savings are required for profitability of a comparable merger under homogeneous Cournot competition. Cost savings equal to $CMCR$ result in zero price increase regardless of demand or cost curvature,[25] for both differentiated and homogeneous Cournot.

[25]The exception is the small price change predicted under loglinear demand and differentiated Cournot competition,

	Differentiated Cournot	Homogeneous Cournot
Firm	$CMCR$	$CMCR$
1	12.02%	25.49%
2	12.72%	25.49%

Table 4: $CMCR$ under both differentiated and homogeneous Cournot competition.

5.4 Comparison to Bertrand models

As summarized in section 1, a literature starting roughly with Singh and Vives (1984) compares outcomes under differentiated Bertrand and Cournot competition, generally finding that Cournot competition is more monopolistic than Bertrand competition. To my knowledge, the literature has not studied whether this intuition extends to merger price effects. That is, if Cournot outcomes are more monopolistic than Bertrand outcomes, should we expect larger merger price effects under Cournot or Bertrand competition?

This question is complicated by the fact that mode of competition is, itself, determinative of calibrated demand. While Singh and Vives (1984) and other papers generally compare Cournot and Bertrand outcomes for the same demand curve, practitioners often start with a set of observables and fit demand to those observables using the various firms' optimality conditions. Since optimality conditions depend on the mode of competition, the same observables produce different demand curves depending on whether competition is thought to be Bertrand or Cournot. Thus, the question I ask here is: for a set of observables, does the demand curve implied by Cournot competition imply greater or lesser merger effects than that implied by Bertrand competition?

For the industry described in table 1, I additionally calibrate a Bertrand demand system, simulate merger outcomes under that system, and simulate merger price effects .[26] Table 6 contains the results. For each type of assumed demand and cost curvature, the calibrated Cournot model predicts a greater merger price effect than does the calibrated Bertrand model.

As a general matter, Cournot merger price effects may be larger or smaller than Bertrand merger price effects. Consider the special case of two identical (or nearly identical) firms producing homoge-

which appears to result from a slight imprecision in the numerical algorithm I use for solving the post-merger equilibrium and the sensitivity of loglinear demand to small changes.

[26]The Bertrand calibration is identical to the Cournot calibration described in section 4.1, but substitutes the Bertrand first order condition $\tilde{m}_i^{Bertrand} = -\frac{1}{\frac{\partial q_i}{\partial p_i}} \frac{q_i}{p_i}$ for than the Cournot condition $\tilde{m}_i^{Cournot} = \frac{\partial p_i}{\partial q_i} \frac{q_i}{p_i}$. Direct demand coefficients b_{ij} are chosen to solve the minimization problem in equation (15), with $\tilde{m}_i^{Bertrand}$ replacing the corresponding Cournot value. Standard techniques allow computation of Bertrand merger price effects, for a variety of cost curves.

		Differentiated Cournot				Homogeneous Cournot	
		Linear demand			Loglinear demand	Linear demand	
Firm	Cost savings	Linear costs	Merging quadratic	Quadratic costs	Linear costs	Merging quadratic	Quadratic costs
1	2%	2.6%	1.53%	1.77%	23.26%	1.53%	3.43%
2	2%	4.09%	1.11%	1.29%	6.18%	1.53%	3.43%
Merger profitability		4.18%	0.76%	1.19%	-22.81%	-3.32%	0.35%
1	6%	1.56%	0.95%	1.1%	13.88%	1.3%	2.92%
2	6%	2.57%	0.71%	0.82%	2.91%	1.3%	2.92%
Merger profitability		16.87%	3.29%	3.57%	-2.53%	-0.46%	2.75%
1	12.02%	0.0%	0.0%	0.0%	0.34%	0.93%	2.07%
2	12.72%	0.0%	0.0%	0.0%	-0.18%	0.93%	2.07%
Merger profitability		39.29%	7.58%	7.59%	38.80%	4.38%	6.76%
1	25.49%	-3.52%	-2.26%	-2.6%	-19.25%	0.0%	0.0%
2	25.49%	-4.85%	-1.61%	-1.86%	-6.84%	0.0%	0.0%
Merger profitability		94.17%	17.76%	17.03%	75.30%	-4.7%	-0.8%

Table 5: Simulated price effects (blue rows) and merger profitability (peach rows) for firms 1 and 2, for various values of merger cost savings realized by the merging firms.

neous (or nearly homogeneous) goods. When separately owned, if the firms compete in price, the textbook Bertrand model predicts they will price at cost (or nearly at cost). In contrast, Cournot duopolists producing identical goods earn a markup above cost, such that each firm has margin $m_i = \frac{s_i}{\epsilon}$, where ϵ is the market elasticity of demand. Following a merger, a monopolist will set price and quantity independently of whether pre-merger competition was Cournot or Bertrand. Since price was lower under the calibrated Bertrand demand system, it follows that the merger price effect is larger under Bertrand competition than under Cournot competition. A more general assessment of Bertrand and Cournot merger price effects is left for future research.

5.5 Capacity constraints

A small literature, notably including Froeb, Tschantz, and Crooke (1999) and Greenfield and Sandford (2021), argues that merger price effects are likely to be attenuated should one or both merging firms be capacity-constrained prior to merging, and likely to be amplified should one or more nonmerging firms be capacity-constrained. The literature further suggests that capacity constraints on merging firms are more important determinants of merger price effects than are capacity constraints on non-merging firms. These papers generally consider differentiated Bertrand models.

I extend these papers to differentiated Cournot competition by examining results in tables 3 and 5. As Greenfield and Sandford (2021) discuss, any capacity constraint can be thought of as an increasing

	Differentiated Bertrand			Differentiated Cournot		
	Linear demand		**Loglinear demand**	**Linear demand**		**Loglinear demand**
Firm	Linear costs	Quadratic costs	Linear costs	Linear costs	Quadratic costs	Linear costs
1	2.8%	1.7%	7.9%	3.1%	2.1%	28.5%
2	3.0%	1.1%	6.8%	4.9%	1.5%	7.7%
3	0.9%	0.7%	0.0%	1.5%	1.0%	0.0%
4	0.8%	0.7%	0.0%	1.5%	1.1%	0.0%
5	0.7%	0.9%	0.0%	1.5%	1.1%	0.0%

Table 6: Cournot calibration predicts a greater merger price increase than Bertrand calibration for each assumed demand and cost structure.

marginal cost function. Thus, the effects of capacity constraints on merger price effects can be seen by comparing merger effects under quadratic costs (constrained) to those under linear costs (unconstrained). Tables 3 and 5 suggest that results on the effect of capacity constraints on merger price effects extend to differentiated Cournot competition. For example, in table 3, merging firms raise price by 3.1% and 4.9% if all firms are unconstrained (linear costs); by 1.5% and 1.1% if merging firms (but not nonmerging firms) are constrained; and by 1.8% and 1.3% if all firms are constrained.

6 Conclusion

The differentiated Cournot model seems to have fallen out of favor with academic researchers, and seems to have never caught on at all with antitrust practitioners. Indeed, the latter group appear to default to competition in prices when products are differentiated, and seem to apply the Cournot model only when goods are thought to be reasonably homogeneous. This lack of interest in the differentiated Cournot model is unfortunate; as discussed in the introduction, both the academic literature and antitrust practitioners view competition in quantity as an important phenomenon, given the prevalence of academic papers and antitrust litigation premised on the (homogeneous) Cournot model. I do not see a theoretical basis for presuming that competition in quantities ceases to be important once products are differentiated.

Moreover, the homogeneous Cournot model is, by design, inflexible. It assumes that all products are interchangeable, which implies that each firm's profit margin is proportional to its market share; in practice, observed shares rarely follow this dictum. The homogeneous Cournot model's assumption of interchangeability of the different goods implies that nonmerging firms are incentivized to increase production following a merger, suggesting that these nonmerging firms may realize the bulk of the benefits of a merger, and indeed merging firms may have greater total profits when separately owned. Finally, the assumption of interchangeability itself is often empirically suspect, as even commodity products are commonly differentiated by branding, idiosyncratic customer preferences, distance from customer locations to manufacturing plant, and minor differences in quality. Each of these inflexibilities opens the Cournot model to attack when used in academic studies or in antitrust litigation.

The differentiated Cournot model circumvents each of the listed inflexibilities associated with the homogeneous Cournot model, while preserving the (presumably important) setting in which firms compete by choosing quantities, with prices set so as to equate each firm's demand and supply. Perhaps the differentiated Cournot model has fallen into disuse because of technical difficulties associated with applying it to industrial organization and antitrust settings. If so, hopefully this paper will give the differentiated Cournot model new life. As described above, I derive a $CMCR$ metric depending only on pre-merger margins, prices, and demand slopes, and which is invariant to assumptions on demand and cost curvature. I show that if the required demand slopes cannot be measured econometrically, they can be calibrated from pre-merger information on shares and diversions, again without relying on assumptions about demand and cost curvature. Finally, should a researcher or practitioner wish to model merger price effects or other counterfactuals, she can use the calibrated demand slopes to populate the parameters of a system of demand and cost curves.

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