Employer discrimination and market structure: Does more concentration mean more discrimination?

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Online appendix

1 Appendix I: Derivation of Aggregate Demand

The consumer's objective is to maximize their expected lifetime utility given by (1). Given that there is no storage technology in the model, and international borrowing and lending is not allowed, consumers spend each period's income m entirely on consumption goods. Consumer i will solve the following static problem every period:

$$\begin{aligned} \max \quad & c_0 + log\left(\int_0^n y(j)^\rho dj\right)^{1/\rho} \\ s.t. \quad & c_0 + \int_0^n p(j)y(j)dj = m \\ & c_0 \geq 0 \end{aligned}$$

Assuming that m is sufficiently large so that $c_0 > 0$, the following conditions must hold:

$$\lambda = 1 \tag{1}$$

$$y(j)^{\rho-1} = \frac{\lambda p(j)}{\int_0^n y(j)^\rho dj} \tag{2}$$

$$c_0 = m - \int_0^n p(j)y(j)dj \tag{3}$$

Note that (2) must hold for each variety j. Indeed, multiplying each side of (2) by $y(j)$, integrating over j, and using $\lambda = 1$ yields:

$$\int_0^n y(j)p(j)dj = \frac{\int_0^n y(j)^\rho dj}{\int_0^n y(j)^\rho dj} = 1 \tag{4}$$

Thus, the per-capita expenditure on the continuum of differentiated product goods is equal to 1 and any excess income is spent on the numeraire good.[1] Since demand is independent of consumer income provided $m > 1$, we suppress discussion of income heterogeneity across individuals and time

[1]This is a standard result given this quasi-linear utlity function although many models introduce a separate demand parameter, μ, to capture the expenditure on differentiated product goods. In this paper, without loss of generality, we have set $\mu = 1$.

in the paper. Next, taking the ratio of condition (2) for varieties j_1 and j_2 and using $\sigma = 1/(1-\rho) > 1$ yields:

$$\frac{y(j_1)}{y(j_2)} = \left(\frac{p(j_1)}{p(j_2)}\right)^{-\sigma}$$
$$y(j_1) = y(j_2)\left(\frac{p(j_1)}{p(j_2)}\right)^{-\sigma}$$

Multiplying both sides by $p(j_1)$ and taking the integral with respect to j_1 yields:

$$\int_0^n y(j_1)p(j_1)dj_1 = \int_0^n y(j_2)p(j_1)^{1-\sigma}p(j_2)^{\sigma}dj_1$$

Using (4), this reduces to:

$$1 = y(j_2)p(j_2)^{\sigma}\int_0^n p(j_1)^{1-\sigma}dj_1$$
$$\Rightarrow y(j_2) = \frac{p(j_2)^{-\sigma}}{\int_0^n p(j_1)^{1-\sigma}dj_1}$$

Aggregating over all E consumers it is direct to derive aggregate demand for a variety given by (3).

1.1 Derivation of HHI as a function of model parameters

In this section, we derive expression (23) in the main body, relating the industry's Herfindahl-Hirschman Index (HHI) to model parameters. First, the HHI at time t is the sum of squared shares across all firms in the industry:[2]

$$HHI(t) = \int_0^{n(t)} s_j(t)dj \tag{5}$$

$s_i(t) = \frac{y_i(t)}{\int_0^{n(t)} y_j(t)dj}$ is firm i's share, or its quantity $y_i(t)$ as a fraction of total quantity. Equations (3) and (11) in the main body of the text yield the following:

$$y_i(t) = \frac{p_i^{-\sigma}E}{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(c_N^{1-\sigma}n_N(t) + c_D^{1-\sigma}n_D(t)\right)} \tag{6}$$

From equation (7) in the main body of text, firms price at a constant markup, given by $p_i = \frac{\sigma}{\sigma-1}c_i$. Substituting this expression for price into the numerator of (6) and simplifying yields:

$$y_i(t) = \frac{Ec_i^{-\sigma}}{\frac{\sigma}{\sigma-1}\left(c_N^{1-\sigma}n_D(t) + c_D^{1-\sigma}n_D(t)\right)} \tag{7}$$

[2]See section 5.3 of U.S. Department of Justice and Federal Trade Commission, *Horizontal Merger Guidelines* (2010), available at https://www.justice.gov/atr/horizontal-merger-guidelines-08192010.

The model's symmetry implies that $y_i(t) = y_D(t)$ for all discriminatory firms and $y_i(t) = y_N(t)$ for all non-discriminatory firms. Hence,

$$\int_0^{n(t)} y_j(t)dj = \frac{\overline{n}_N E c_N + \overline{n}_D E c_D}{\frac{\sigma}{\sigma-1}\left(c_N^{1-\sigma} n_D(t) + c_D^{1-\sigma} n_D(t)\right)} \tag{8}$$

Dividing the right hand side of (7) by that of (8) yields an expression for the share of each discriminatory and non-discriminatory firm as a function of model parameters:

$$s_D = \frac{\phi^{-\sigma}}{\overline{n}_N + \overline{n}_D \phi^{-\sigma}} \tag{9}$$

$$s_N = \frac{1}{\overline{n}_N + \overline{n}_D \phi^{-\sigma}} \tag{10}$$

Plugging (9) and (10) into the expression defining HHI, equation (5), yields:

$$\begin{aligned} HHI &= \overline{n}_N s_N^2 + \overline{n}_D s_D^2 \\ &= \frac{\overline{n}_N + \overline{n}_D \phi^{-2\phi}}{(\overline{n}_N + \overline{n}_D \phi^{-\phi})^2} \end{aligned} \tag{11}$$

Equation (11) appears in the text as equation (23).